

Multi-Terminal Coulomb-Majorana Junction

Alexander Altland¹ and Reinhold Egger²

¹*Institut für Theoretische Physik, Universität zu Köln, Zùlpicher Str. 77, D-50937 Köln, Germany*

²*Institut für Theoretische Physik, Heinrich-Heine-Universität, D-40225 Düsseldorf, Germany*

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We study multiple helical nanowires in proximity to a common mesoscopic superconducting island, where Majorana fermion bound states are formed. We show that a weak finite charging energy of the center island may dramatically affect the low-energy behavior of the system. While for strong charging interactions, the junction decouples the connecting wires, interactions *lower* than a non-universal threshold may trigger the flow towards an exotic Kondo fixed point. In either case, the ideally Andreev reflecting fixed point characteristic for infinite capacitance (grounded) devices gets destabilized by interactions.

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Introduction.—Electronic transport through topological insulators [1] or superconductors [2] has come into the limelight of condensed-matter physics over the past few years. In particular, understanding the physics of localized Majorana bound states, generically expected near the ends of one-dimensional (1D) topological superconductor wires [3–5], is crucial for exploiting the non-Abelian statistics of Majorana fermions in topological quantum computation schemes [6–8]. When a grounded Majorana nanowire — such as InSb and InAs wires subject to a Zeeman field and proximity coupling to an *s*-wave superconductor [9, 10] — is weakly contacted by a normal metal, the presence of a Majorana state should reflect in a conductance peak [11–15], and signatures of this type were indeed observed experimentally [16–19].

In this paper, we discuss novel transport phenomena caused by Coulomb interactions in devices comprising Majorana wires contacted to leads. Our setup is sketched in Fig. 1: For N nanowires proximity-coupled to the same mesoscopic superconducting island, there are $2N$ Majorana fermions, $\gamma_j = \gamma_j^\dagger$, with anticommutator $\{\gamma_j, \gamma_k\} = \delta_{jk}$. The island connects to the j th lead by tunneling through the Majorana state γ_j with coupling strength t_j ; all other coupling mechanisms are irrelevant [20]. Coulomb interactions affect the system in two distinct ways: (i) The combination of repulsive interactions, spin-orbit coupling and Zeeman field is expected to turn each of the $M \leq 2N$ *connecting leads* into a spinless (helical) Luttinger liquid (LL) characterized by an interaction constant $g < 1$ [21–23]. (ii) Weak intra-island interactions do not compromise the integrity of individual Majoranas [24–26]. However, they introduce correlations *between* these states, and thereby correlations between the connecting wires. It stands to reason that the option to generate inter-wire coupling on mesoscopic scales, i.e., independently of microscopic single-particle tunneling between distinct terminals [27, 28], will be crucial for quantum computation (and other) applications.

Our main observations are summarized as follows. It turns out that E_c , no matter how weak, destabilizes the

fully Andreev reflecting fixed point characterizing the grounded system [6]. Thinking of tunneling events from and into lead j in terms of particles and anti-particles, charging generates a strong confinement force operational at time scales $\tau \gtrsim E_c^{-1}$. At larger time scales, tunneling events are bound into overall charge neutrality preserving ‘dipoles’, describing the near instantaneous (at scales E_c^{-1}) transmission of charge from lead j to lead $k \neq j$. The formation of these objects terminates the up-renormalization of coupling constants prevalent in the ‘asymptotic freedom’ regime $\tau \lesssim E_c^{-1}$. At scales $\tau \gtrsim E_c^{-1}$, the effective tunneling strengths are subject to a competition of downward renormalization due to fluctuations of individual dipoles, and upward renormalization due to dipole-dipole interactions. The balance of these two mechanisms defines a repulsive fixed point, λ^* , separating a flow towards the decoupled dot (zero coupling) from a flow towards an exotic Kondo regime, generalizing the $M = 3$ topological Kondo effect discussed in Ref. [29]. Whether the system ends up in the trivial weak-coupling limit, or in the Kondo regime, depends on the bare coupling constants as much as on the value of E_c : For *weak* charging, the initial growth of the bare charge coupling constants persists up to larger time scales, which gives the system an enhanced chance to reach couplings $> \lambda^*$ before the confinement regime takes over. We finally note that in supporting a novel type of Kondo flow, the present system differs strongly from the $M = 2$ Majorana single-charge transistor [30–32] as well as from other types of previously studied LL junctions [33–35].

Model.—We start by deriving the low-energy effective theory for the setup in Fig. 1. The Hamiltonian, $H = H_c + H_t + H_l$, contains a piece H_c describing the island, the tunnel Hamiltonian H_t , and H_l for the LL leads (we put $\hbar = k_B = 1$ below). For each nanowire ($\alpha = 1, \dots, N$), we have two spatially well separated Majorana fermions, $\gamma_{2\alpha-1}$ and $\gamma_{2\alpha}$. It is useful to define the nonlocal auxiliary fermion operators $d_\alpha = (\gamma_{2\alpha-1} + i\gamma_{2\alpha})/\sqrt{2}$, where the total Majorana occupation number operator is $\hat{n} = \sum_\alpha d_\alpha^\dagger d_\alpha$. Assuming that the proximity gap ex-

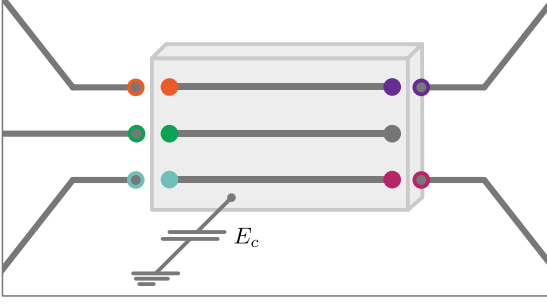


Figure 1. (Color online) Schematic setup of the multi-terminal Coulomb-Majorana junction. N helical nanowires (here, $N = 3$) connect to a floating mesoscopic superconductor (shown as gray box) with charging energy E_c . Majorana fermions γ_j (filled circles) exist near the ends of proximity-coupled wire segments. The remaining wire parts (away from the island) act as LL leads connected to the island by tunnel couplings t_j . Majorana fermions representing Klein factors are shown as open circles. The case of $M < 2N$ leads (here, $M = 5$) is included by putting one or several $t_j = 0$. Direct tunneling between the γ_j is neglected.

ceeds all other energy scales, the island is fully described in terms of the γ_j and the Cooper pair number operator, \hat{N}_c , or conjugate phase, φ , where $[\varphi, 2\hat{N}_c] = i$. Since both, the Majoranas, and the superconducting phase are zero-energy modes, the island Hamiltonian is solely due to charging effects,

$$H_c = E_c(2\hat{N}_c + \hat{n} - n_g)^2, \quad (1)$$

where n_g is a dimensionless gate voltage parameter. The semi-infinite LL leads, with tunnel contacts at $x = 0$, are described by dual bosonic fields, $\phi_j(x)$ and $\theta_j(x)$ [21], where

$$H_l = \frac{v_F}{2\pi} \sum_{j=1}^{2N} \int_0^\infty dx [g(\partial_x \phi_j)^2 + g^{-1}(\partial_x \theta_j)^2] \quad (2)$$

with Fermi velocity v_F . The fermion annihilation operator for a right- or left-mover reads $\psi_{j,R/L}(x) = a^{-1/2} \eta_j e^{i(\phi_j \pm \theta_j)}$, where a is a short-distance cutoff and the η_j are additional Majorana fermions with $\{\eta_j, \eta_k\} = \delta_{jk}$ [22]. These ‘Klein factors’ ensure fermion anticommutation relations between different leads and play an important role below. Since H_l describes decoupled leads with perfect normal reflection at $x = 0$, we impose open boundary conditions, $\psi_{j,L}(0) = \psi_{j,R}(0)$, pinning all fields $\theta_j(0)$. A lead fermion operator near the respective tunnel contact is thus given by $\Psi_j = a^{-1/2} \eta_j e^{i\phi_j(0)}$. Finally, the tunnel Hamiltonian reads [32]

$$H_t = \sum_j \tilde{t}_j \Psi_j^\dagger \left(d_{[j/2]} + (-)^{j-1} e^{-2i\varphi} d_{[j/2]}^\dagger \right) + \text{H.c.}, \quad (3)$$

where $\tilde{t}_j = (a/2)^{1/2} t_j$. Tunneling either leaves no trace in the condensate (the first term), or must involve the oper-

ators $e^{\pm 2i\varphi}$, raising or lowering the Cooper pair number N_c by one unit.

Effective phase action.—Next we derive an equivalent Euclidean phase action, S , which will allow us to better understand the physics. We start by integrating over the harmonic LL bosons away from $x = 0$, retaining only $\Phi_j(\tau) \equiv \phi_j(0, \tau)$. This generates the action piece $S_l = \frac{Tg}{2\pi} \sum_{j,n} |\omega_n| |\Phi_j(\omega_n)|^2$, with Matsubara frequencies $\omega_n = 2\pi nT$ for temperature T ($n \in \mathbb{Z}$). Next, by virtue of a Hubbard-Stratonovich transformation to the condensate phase field $\varphi(\tau)$, the charging term in Eq. (1) leads to the contribution

$$S_c = 2\pi i W n_g + \int_0^{1/T} d\tau \left(\frac{\dot{\varphi}^2}{4E_c} - i\dot{\varphi} \hat{n} \right). \quad (4)$$

The phase field has the periodicity $\varphi(\tau + 1/T) = \varphi(\tau) + 2\pi W$ with winding number $W \in \mathbb{Z}$. However, below we mainly focus on the $W = 0$ sector, which is appropriate for weak-to-intermediate charging energy. Noting that there also is a free fermion piece, $S_f = \sum_\alpha \int d\tau \bar{d}_\alpha \dot{d}_\alpha + \frac{1}{2} \sum_j \int d\tau \eta_j \dot{\eta}_j$, we now perform a gauge transformation, $d_\alpha(\tau) \rightarrow e^{-i\varphi(\tau)} d_\alpha(\tau)$, in order to eliminate the $\dot{\varphi} \hat{n}$ term in Eq. (4). It is then convenient to switch back from the d fermions to Majoranas, $\gamma_{2\alpha-1} = (d_\alpha + d_\alpha^\dagger)/\sqrt{2}$ and $\gamma_{2\alpha} = -i(d_\alpha - d_\alpha^\dagger)/\sqrt{2}$. After these steps, the tunnel action follows as

$$S_t = \sum_j t_j \int d\tau (-2i\eta_j \gamma_j) \sin(\Phi_j + \varphi). \quad (5)$$

At this stage it is useful to note that for each j , the Klein factor η_j and the Majorana γ_j can be fused to form the auxiliary fermion $f_j = (\eta_j - i\gamma_j)/\sqrt{2}$, where $-2i\eta_j \gamma_j = 2f_j^\dagger f_j - 1 \equiv \sigma_j = \pm$. The resulting fermion Hamiltonian conserves $f_j^\dagger f_j$, and thus also each parity number σ_j . Hence $\sigma_j = \pm$ is not a dynamical variable but just has to be summed over in the functional integral [36]. The sole effect of this summation is to eliminate contributions of odd order in the tunneling amplitudes t_j to the perturbative expansion of the functional integral, while even-order contributions remain unaffected. Keeping only even orders, we will drop the then immaterial presence of the σ_j ’s throughout. This Klein-Majorana fusion procedure implies an enormous technical simplification, since we are now left with an effective phase action only. Before writing down this action, we shift $\Phi_j \rightarrow \Phi_j - \varphi$, which removes φ from the tunnel action (5). Performing the remaining Gaussian functional integral over φ , after unitary transformation to the discrete Fourier basis ($q = 0, \dots, 2N - 1$),

$$\tilde{\Phi}_q(\tau) = \frac{1}{\sqrt{2N}} \sum_{j=1}^{2N} e^{i\pi qj/N} \Phi_j(\tau), \quad (6)$$

we arrive at the effective phase action

$$S = \frac{Tg}{2\pi} \sum_{q,n} \frac{|\omega_n|}{1 + \delta_{q,0} \epsilon/|\omega_n|} \left| \tilde{\Phi}_q(\omega_n) \right|^2 \quad (7)$$

$$+ \sum_j t_j \int d\tau \sin \Phi_j(\tau),$$

with the energy scale $\epsilon \equiv 4gNE_c/\pi$. We stress that the charging energy only affects the zero mode $\tilde{\Phi}_0$, which effectively becomes *free* at low frequencies, $|\omega_n| \ll \epsilon$.

Particle analogy and renormalization.—We next turn to discussing the perturbative expansion in the tunnel couplings t_j . A very instructive analogy follows by interpreting $\mathcal{O}_j^\pm(\tau) = e^{\pm i\Phi_j(\tau)}$ as particles (‘quarks’) and antiparticles living on the imaginary-time axis. Our particles carry a ‘flavor’ index ($j = 1, \dots, 2N$) and an (imaginary-valued) effective charge $-it_j/2$. We study the properties of the ensuing interacting particle gas by standard renormalization group (RG) methods [23, 37]. In an RG step, we integrate over fast Φ_j modes with frequencies in the shell $\Lambda/b < |\omega_n| < \Lambda$, with rescaling parameter $b > 1$ and a high-energy cutoff Λ of the order of the proximity gap. The RG step is completed by rescaling all frequencies, $\omega \rightarrow b\omega$, such that Λ stays invariant. For small t_j , the particle density is low and different charges (i.e., the t_j) renormalize independently. We first RG-integrate out modes in the shell $\epsilon < |\omega_n| < \Lambda$, where the zero mode $\tilde{\Phi}_0$ is not significantly affected by E_c , see Eq. (7). Some algebra yields the net scaling dimension $1 - 1/(2g)$ for t_j , which are therefore RG-relevant couplings for $g > 1/2$. This signals a flow towards the resonant Andreev reflection fixed point [20]. In the lead-non-interacting case, $g = 1$, the scaling dimension $1/2$ represents the naive dimension of a fermion-Majorana scattering operator. After the RG integration over the shell $\epsilon < |\omega| < \Lambda$, the renormalized couplings are then given by $t_j^{(1)} = t_j(\Lambda/\epsilon)^{1-1/(2g)}$.

Proceeding to lower frequency scales, $|\omega| < \epsilon$, the charging energy renders the zero-mode action non-dissipative, $S[\tilde{\Phi}_0] \simeq \frac{Tg}{2\pi\epsilon} \sum_n \omega_n^2 |\tilde{\Phi}_0(\omega_n)|^2$, see Eq. (7). The integration over $\tilde{\Phi}_0$ generates a strong ‘confinement’ potential linear in the time separation between particle creation, $\mathcal{O}_j^+(\tau) \sim e^{+i\tilde{\Phi}_0(\tau)/\sqrt{2N}}$, and particle annihilation events, $\mathcal{O}_k^-(\tau') \sim e^{-i\tilde{\Phi}_0(\tau')/\sqrt{2N}}$ of arbitrary flavor index $k \neq j$. (For $k = j$, particle-anti-particle annihilation occurs – the inconsequential virtual tunneling of a particle to and fro the same lead, cf. Fig. 2 upper inset.) At time resolutions lower than E_c^{-1} , the relevant excitations of the theory then are quark-antiquark pairs (‘mesons’) [38],

$$\langle \mathcal{O}_j^+(\tau) \mathcal{O}_k^-(\tau') \rangle_0 \simeq e^{-\frac{2E_c}{\pi} |\tau - \tau'|} \mathcal{O}_{jk}(\tau), \quad (8)$$

$$\mathcal{O}_{jk}(\tau) = e^{i\Phi_j(\tau)} e^{-i\Phi_k(\tau)}.$$

RG equations in dipole regime.—We now consider the effective low-energy theory at resolutions $E_c\tau > 1$, where

we are dealing with a gas of dipoles, \mathcal{O}_{jk} , with couplings λ_{jk} , $j \neq k$. The ‘bare value’ of these couplings is given by $\lambda_{jk}^{(1)} \approx t_j^{(1)} t_k^{(1)} / E_c > 0$, where the factor E_c^{-1} is due to a time integration over the distance between particle and antiparticle. Physically, these couplings bundle the effect of in-tunneling from lead j into a virtual on-dot state of longevity $\sim E_c^{-1}$, followed by out-tunneling into lead k . We then continue the frequency-shell RG integration for mesons, going up to one-loop accuracy. Some algebra yields the RG equations

$$\frac{d\lambda_{jk}}{d \ln b} = (1 - g^{-1}) \lambda_{jk} + \frac{\kappa}{E_c} \sum_{m \neq (j,k)} \lambda_{jm} \lambda_{mk}, \quad (9)$$

where κ is a non-universal constant of order unity. The first term describes the standard power-law suppression of the tunneling density of states in/out of the LL leads [21] and implies a suppression of the λ_{jk} . This term is fought by the positive second contribution, which describes the effect of dipole fusion $[(j, m) + (m, k) \rightarrow (j, k)]$ due to dipole-dipole interactions (cf. Fig. 2 lower inset.) Equation (9) suggests the existence of an RG-unstable fixed point with isotropic couplings,

$$\lambda_{jk} = \lambda^* = \frac{g^{-1} - 1}{\kappa(M - 2)} E_c. \quad (10)$$

Discussion.—When decreasing the effective frequency scale ω , typical couplings, t and λ , show the RG flow illustrated in Fig. 2. During the initial RG flow (down to $\omega \approx E_c$), particles are asymptotically free, implying a power-law increase of a typical tunnel coupling, $t \propto \omega^{\frac{1}{2g}-1}$, with the time-like variable ω^{-1} . We now compare the value $\lambda^{(1)} = (t^{(1)})^2 / E_c \sim E_c^{-3+1/g}$, reached at the end of the asymptotic freedom phase ($\omega \approx E_c$), to the fixed-point value $\lambda^* \sim E_c$ in Eq. (10). For sufficiently large E_c [39], the fixed point cannot be reached, and therefore the $\omega \rightarrow 0$ stable fixed point describes a completely decoupled LL junction. For sufficiently small E_c , however, the fluctuations generated during the asymptotic freedom phase tip the balance, $\lambda^{(1)} > \lambda^*$. A straightforward expansion near the fixed point then shows that the isotropic baseline $\bar{\lambda} > \lambda^*$ of the couplings flows to large values with dimension $g^{-1} - 1 > 0$, while deviations between the couplings are RG irrelevant, i.e., the flow is towards an isotropic configuration $\lambda_{jk} = \bar{\lambda}$. At a characteristic ‘Kondo temperature’, $T_K \sim E_c \exp\left(-\frac{E_c}{\lambda^{(1)} \kappa (M-2)}\right)$, the coupling $\bar{\lambda}$ begins to diverge, and our perturbative expansion is no longer applicable. A comprehensive discussion of the ensuing strong-coupling regime is beyond the scope of the present paper. However, by analogy to the $M = 3$ topological Kondo fixed point [29], we expect that $\bar{\lambda} \rightarrow \infty$ is the only fixed point besides $\bar{\lambda} = 0, \lambda^*$.

Conductance matrix.—The two-stage scenario outlined above, with its branching into either a strong-coupling or

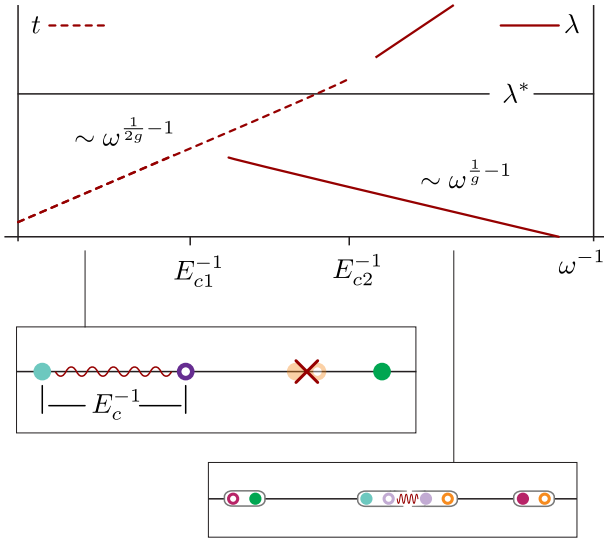


Figure 2. (Color online) Schematic RG flow of typical couplings, t and λ , vs the time-like RG parameter ω^{-1} , and illustration of the system excitations. At short times, individual in- and out-tunneling events into different leads dominate (indicated by dots and circles of different color). These ‘quarks’ are asymptotically free, and their fluctuations generate a power-law increase of t . For $\omega \lesssim E_c$, confinement leads to bound quark-antiquark dipoles (mesons) for $j \neq k$, or annihilation for $j = k$. The dipole coupling, λ , experiences a downward renormalization due to individual fluctuations and an upward renormalization due to dipole-dipole fusion events (see also right lower inset). For sufficiently large $E_c = E_{c1}$, λ does not reach the fixed-point value (10), the first mechanism dominates, and hence λ flows to the decoupled fixed point as $\omega \rightarrow 0$. For small $E_c = E_{c2}$, however, the increase of t during the asymptotic freedom phase brings λ beyond the balance point λ^* (see text), and the RG flow proceeds towards the strong-coupling topological Kondo fixed point.

a decoupled low-frequency limit, will bear consequences for all physical observables. We here briefly discuss the resulting temperature dependence of the linear conductance G_{jk} between different leads j and k . For high $T > E_c$, asymptotic freedom causes the power-law scaling $G_{jk} \sim T^{-2+1/g}$, while for $T \rightarrow 0$, we either have a vanishing conductance, $G_{jk} \sim T^{-2+2/g}$, if the decoupled fixed point is approached, or the Kondo-type flow approaching the unitary limit of an isotropically hybridized junction, $G_{jk} = \frac{2e^2}{h} \frac{1}{M}$. The latter case constitutes the M -terminal generalization of the teleportation scenario discussed for $M = 2$ by Fu [31]. It is worth stressing that for both stable fixed points (Kondo or decoupled), resonant Andreev reflection is unstable, and hence arbitrary E_c destroy the corresponding fixed point.

Conclusions.—In this paper, we have formulated and studied the problem of junctions of Majorana wires meeting on a superconducting island with charging energy E_c . We also included correlations in the leads, since these typically are 1D nanowires themselves. Our RG analysis

reveals that the physics of the system is determined by asymptotic freedom at short time scales ($\tau < E_c^{-1}$) and confinement at long time scales ($\tau > E_c^{-1}$) corresponding, respectively, to independent dot-lead tunneling, and virtual co-tunneling. For sufficiently weak E_c , we find that a strong-coupling Kondo fixed point is approached, while otherwise the junction describes decoupled leads at $T = 0$. These predictions can be tested using the temperature dependence of the conductance.

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